IPART
New South Wales

Independent Pricing and Regulatory Tribunal

Adjusting for expected inflation in deriving the cost of capital

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Invitation for submissions

IPART invites written comment on this document and encourages all interested parties to provide submissions addressing the matters discussed.

Submissions are due by 9 April 2009.

We would prefer to receive them by email <wacc@ipart.nsw.gov.au>.

You can also send comments by fax to (02) 9290 2061, or by mail to:

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If you would like further information on making a submission, IPART's submission policy is available on our website.

Contents

1 Introduction and Summary

This paper discusses the possible approaches to adjusting the cost of capital for inflation and proposes a preferred alternative for the consideration and response of stakeholders.

IPART uses a real rate of return in setting prices to cover a utility's costs. This requires the conversion of a nominal cost of capital to a real cost of capital by adjusting for expected inflation. Under the CPI-X formula prices are then adjusted annually to reflect the actual change in inflation.

Currently, IPART uses the difference between nominal and real Commonwealth bond rates to adjust for expected inflation. However, problems have arisen with this approach. In particular:

- the Commonwealth Government has indicated that it will not issue inflation-indexed bonds beyond the 2020 maturity series
- there are concerns that the inflation-indexed bond rate is biased because of the scarcity of indexed Commonwealth bonds.

This has led IPART to investigate alternative financial market instruments which could be used to adjust for expected inflation. IPART engaged Erik Schloegl to prepare a paper (Appendix A) examining the options available. The paper was reviewed by Ian Alexander (Director, Cambridge Economic Policy Associates).

Adjustments for inflation can be based on non-market data, such as economists' forecasts of inflation, or financial market data that reflects the real cost of funds or the cost of inflation hedges. IPART considers that market-based measures are, in principle, superior. To the extent that economists' forecasts differ from the market values they create a gap between the allowed cost of debt, for example, and the cost to the utility of borrowing in real terms. Decisions on financing strategies are a matter for the business not the regulator, but such a gap is, in principle, of some concern.

Erik Schloegl developed two methodologies for adjusting for inflation through the use of market data:

- the first is based on a wider range of inflation-indexed bonds, including bonds issued by State governments, and
- the second is based on inflation-indexed swaps.

Inflation-indexed swaps are financial instruments which allow two parties to exchange inflation-indexed cash flows for a cash flow fixed in nominal terms (ie, the same dollar amount in each year). The market for inflation-indexed swaps is quite deep in Europe and the US. The market for inflation-indexed swaps developed in Australia in the mid-1990s and there are indications that the swap market is becoming more actively used in large transactions, especially for infrastructure financing.

Due to the scarcity of inflation-indexed bonds, Erik Schloegl recommends using inflation-indexed swap market[1]. However, Erik Schloegl also recommends that the swap rates should be compared against the inflation rate derived from government or semi-government bond market data where available.

In principle IPART agrees with Erik Schloegl. Subject to cross-checking against bond rate differentials, it believes that the zero coupon inflation-indexed swap market provides a robust alternative to IPART's current methodology of estimating a 10 years inflation rate.

IPART seeks comments on the following

1 What are the advantages and disadvantages of basing the adjustment for the inflation rate on either financial market data or on economists' forecasts?

2 If a financial market based forecast is favoured, which financial market instruments, for example bonds, swaps or others, should be used?

3 Should volatility in the zero coupon inflation-indexed swap market be addressed by using a 20-day average of quotes (consistent with the current approach)? Alternatively, should different periods of averaging or no averaging at all be used?

4 Are transaction costs appropriately reflected by using the midpoint of the bid/offer spread?

5 Any other comments on Erik Schloegls' proposed swap market methodology set out in the accompanying consultant's report.

Submissions should be provided by 9 April 2009.

Table 1.1 Review timetable

Date	Event
9 April 2009	Public submissions due
8 May 2009	Release of final decision

1 See appendix A.

2 Framework for adjusting inflation

This chapter explains the adjustments for inflation that must be made under IPART's approach to regulation and the framework used for estimating the cost of capital. It provides the context for the subsequent assessment of alternative approaches to adjusting for inflation.

2.1 The importance of inflation in IPART's determinations

In determining the allowed revenues for the regulated businesses, the regulator has to compensate the firm for (amongst other things) the effect of inflation. IPART achieves this by using a CPI-X approach to setting prices that is based on providing for a "real return on real asset base". Under this approach adjustments for inflation are required to:

1. Convert the nominal cost of capital to a real cost of capital.
2. Index the regulatory asset base by forecast inflation in order to calculate the required return on the assets and return of the assets (depreciation).
3. Convert all other costs and revenues to nominal terms and calculate the X factor in the CPI-X formula.

The objective of the first adjustment is to provide a measure of the real cost of funds for the utility. Ideally it should be possible for an efficient, financially sound utility to match the assumptions underlying the estimated cost of capital in the market place. Discrepancies between the adjustment for inflation and the costs of the relevant financial market instruments mean that this will not be the case. Put simply if IPART were to adopt economist forecasts of inflation of 2.5 per cent in converting from nominal to a real cost of funds when the implied inflation rate in the financial markets was 2.0 per cent the utility would not be able to borrow in real terms at the interest rate implicit in the regulated WACC. Hence, an important objective for the first adjustment is that it be aligned to market values. Furthermore, the first adjustment has a 10-year horizon as the cost of funds is based on bonds and debt raisings with a 10-year maturity.

In contrast the second and third adjustments have a much shorter horizon. As the price caps are generally set for 4 to 5 years, a 4 to 5 year forecast inflation is required. Furthermore, for these purposes discrepancies between forecast inflation and actual inflation may be of more concern than discrepancies with the implied expectations of inflation in financial markets. The key point is that the one adjustment for inflation may not suit all three purposes.

This paper is concerned with the first of these adjustments. Once IPART has considered the adjustment for this purpose it may need to consider whether this adjustment should also be used for the other purposes.

As noted above, allowed revenues need to provide for the effect of inflation. US regulators use a 'nominal return on nominal asset base' approach. This allows for inflation in the nominal return on assets.[2] However, in common with many other regulators outside the USA and New Zealand, IPART uses a "real return on real asset base" framework. Under this approach the assets are indexed to maintain their value in real terms. Hence, only a real return on assets need be provided.

2.2 What is cost of capital?

Given the capital intensive nature of utility and transport infrastructure, the return on the businesses' regulatory asset base (RAB) is a major – and often controversial – component in determining the regulated price. The cost of capital is the level of return required by investors in order to provide capital to a firm. Risk is caused by the possibility of different outcomes, which results in uncertainty. In determining the appropriate return on capital for a regulated business, the allowed cost of capital should represent the relative riskiness of the regulated business. It is important that cost of capital:

- reflects the opportunity cost of capital to ensure that prices provide a true signal to users of cost of the services provided, and
- provides a commercially sustainable return.

These requirements apply irrespective of whether a real cost of capital or nominal cost of capital is used. As noted above, IPART currently uses a real version of the cost of capital and it uses expected inflation implicit in real and nominal risk-free rates to convert the nominal into a real cost of capital.

2.3 Calculation of cost of capital

In common with many other regulators, IPART calculates the rate of return on the RAB by reference to the weighted average cost of capital (WACC). The WACC of a business is the expected cost of the various classes of its capital (eg, equity and debt), weighted by the proportion of each class of capital to the total capital of the firm. Under this approach the cost of debt is calculated as a margin over the risk-free rate, while the cost of equity is calculated using the capital asset pricing model (CAPM).

2 The nominal return can be decomposed into an allowance for inflation and a real return on assets.

The WACC approach for calculating the cost of capital for a business is widely used and accepted. It is the method adopted by most financial practitioners and remains the preferred methodology of most regulators.

The process of calculating the WACC involves applying a series of equations to determine the weighted average of returns to debt and equity in a given market at a point in time. In principle, the real WACC could be estimated directly from real interest rate data without a conversion from a nominal form. However, the market for indexed securities is not as deep as that for nominal securities[3]. Nominal interest rates and nominal WACCs are more commonly quoted and used in business. In practice regulators have estimated the WACC in nominal terms and converted this to a real WACC using an estimate of inflation expectations.

As noted above IPART uses a real version of the WACC. Hence, the inflation rate is one of the inputs needed for its calculation. IPART's preference is to use a WACC that matches as closely as possible the expected life of the assets being regulated. However, there are few securities traded with the necessary maturity and IPART has used 10-year yields. Hence, IPART uses a 10-year maturity for its inflation rate used to convert the nominal into a real WACC.

The choice of whether a nominal or real WACC is used in the calculation of the cost of capital should not impact on the overall level of return granted; it simply defines the form of regulatory model adopted. The overall cash flows over the life of the assets should be consistent between models.

2.4 IPART's current approach to converting from a nominal to real WACC

Currently IPART calculates the 10-year expected inflation from the difference between the 10-year nominal Commonwealth Government bond (CGS) and the real risk-free rate. This is done by using the Fisher equation[4] to solve for the inflation rate (i) (Box 2.1).

Box 2.1 Fisher equation

$$1+r_{nominal}=(1+r_{real})\times(1+i)$$

[3] Ie, securities whose repayments of principal are not adjusted for inflation.

[4] Brealey, R. and S. Myers, *Principles of Corporate Finance,* fifth edition, New York McGraw–Hill: 1996, pp 642-643.

In principle, the differential should reflect full information available, including the various economic forecasts of inflation implicit in the bond rate difference. Financial market estimates avoid the use of economists' forecasts and have in the past been considered more objective by IPART. Commencing with its decision on the 2008 determination for Sydney Water Corporation, the IPART has adjusted the yield on indexed CGS (r_{real}) by 20 basis points to offset the effect of the scarcity of these securities on their yield (see discussion in next chapter). This reduces the implied inflation expectations by 20 basis points.

It is generally straightforward to obtain a robust estimate of the nominal risk-free rate from Commonwealth bond rate data. However, obtaining a robust estimate of the real risk-free rate has become increasingly difficult. The longest maturity for real bonds available is the 2020 issue. The Australian Office of Financial Management has signalled that it will not issue any further inflation-indexed bond series. This raises two significant issues for IPART:

- If IPART wishes to continue its current approach it would need to either use extrapolation to estimate a 10-year real interest rate or reduce the reference maturity for the estimation of the risk-free rates to, say, 5 years.
- The scarcity of Commonwealth indexed bonds is also likely to become an increasingly important factor for the yields on these securities. IPART therefore currently adjusts the breakeven inflation rate by 20 basis points to account for this relative scarcity. This references estimates by NERA in 2007[5]. If this approach is continued a more thorough examination of the size and stability of the premium is required.

If IPART wishes to continue to use its current approach it would need to use an extrapolation to estimate a 10-year real interest rate or reduce the reference maturity for the estimation of the risk-free rate to, say, 5 years. However, this would only be a solution for an additional 6 years until the 2020 inflation-indexed bond reaches a term-to-maturity of 5 years. IPART notes that semi-government and corporate inflation-indexed bonds do not trade frequently and thus do not currently qualify as an alternative to Commonwealth Government inflation-indexed bonds.

5 NERA, *Bias in inflation –indexed CGS yields as a proxy for the CAPM risk-free rate*, March 2007.

3 Why is IPART reviewing its approach to adjusting for inflation?

IPART considers that there is strong merit in maintaining a consistent approach to the cost of capital across regulatory decisions as it reduces regulatory risk and its associated costs. There is a presumption that unless an alternative approach to the calculation of a WACC parameter is demonstrated to be clearly superior, the existing approach should be preferred. However, two factors – the bias in the yields on indexed government securities and the shortening of the available maturities –will make the continuation of the existing approach more difficult. This chapter outlines these concerns and indicates how other regulators in Australia have responded to these concerns.

3.1 Bias in yields on indexed government securities

NERA[6], the Allen Consulting Group (ACG)[7], the Reserve Bank of Australia (RBA)[8] and the Australian Treasury[9] have all raised concerns regarding a bias in the yields on inflation-indexed bonds.

To test whether there is a bias in the real risk-free rate used by Australian regulators, NERA calculated the margins that the real and nominal corporate bonds were yielding over the equivalent real and nominal government bonds. NERA found that the margin for real corporate bonds became larger than the margin for nominal corporate bonds from the last quarter of 2004 and increased until it reached a value of approximately 20 basis points by March 2007. It concluded that it was because of the relative scarcity of Treasury inflation-indexed bonds in the absence of any new issues by the Commonwealth. NERA noted that the number of Treasury inflation-indexed bonds on issue had fallen by 1/5th (as a proportion of GDP) in the previous 2 years.

NERA considered that both nominal and real government bonds are becoming poor proxies for nominal and real risk-free rates under the CAPM, and that upward adjustments should be made to their yields to correct for the biases.

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6 NERA, *Bias in inflation –indexed CGS yields as a proxy for the CAPM risk-free rate*, March 2007. NERA, *Absolute bias in (nominal) Commonwealth Government Securities*, June 2007.

7 ACG, *Relative bias' of inflation indexed CGS yields as a proxy for the CAPM risk-free rate,* July 2007.

8 RBA, Letter to ACCC, 9 August 2007.

9 Australian Treasury, *The Treasury bond yield as a proxy for the CAPM risk-free rate*, Letter to the ACCC, 7 August 2007.

ACG concluded that there is evidence that the yields on Treasury inflation-indexed bonds result in a downward-biased estimate of the real risk-free rate. The reasons offered for this view by ACG were as follows:

- The forecasts of inflation implied by returns on government bonds are generally above the target inflation range of the RBA of two to three per cent. As at 28 June 2007, the average annual levels of inflation implied by the 2010, 2015 and 2020 inflation-indexed bonds were 2.8 per cent, 3.3 per cent and 3.5 per cent respectively. The level of inflation implied by the 10-year nominal and real risk-free rates calculated using the Fisher equation was 3.3 per cent.
- ACG's consultation with a number of financial market participants on the conditions in the market for Treasury inflation-indexed bonds indicated that most market participants consider that there is an element of downward bias in yields of these bonds.

The RBA and the Australian Treasury's views are that:

- demand for inflation-indexed bonds has increased as supply has fallen
- turnover for these bonds is low and the market is fairly illiquid, and
- as Treasury inflation-indexed bonds mature without replacement, their usefulness for estimating long-term real risk-free rates will diminish.

3.2 What has been done in other jurisdictions?

Some Australian regulators have started looking for other methodologies that can be used to estimate the inflation rate. For example, in early 2008 the AER released its determination on SP Ausnet[10] where it used a methodology that references the RBA's short-term inflation forecasts, which extend out 2 years, and adopted the midpoint of the RBA's target inflation band beyond that period (ie, 2.5 per cent). This resulted in an inflation forecast of 2.6 per cent.

In its final decision of the Gas Access Arrangement Review 2008-2012, the ESC applied an inflation forecast of 2.7 per cent based on a range of considerations including market practice in making assumptions of long-term inflation, levels of historical inflation and the RBA's target range for the rate of inflation[11].

In its 2008 draft determination for NSW electricity distribution[12], the AER used the same approach that it used for the SP Ausnet determination (ie, the RBA's short-term inflation forecast and the mid-point of the RBA's target inflation range beyond that period). In its review of WACC parameters of electricity transmission and distribution, the AER proposes to shorten the term of the interest rates and inflation from 10 to 5 years.

10 AER, *Final Decision: SP Ausnet Transmission Determination 2008/09 to 2013/14*, January 2008, pp 105,106.

11 ESC, *Gas Access Arrangements Review 2008-2012 Final Decision*, March 2008, p 460.

12 AER, *NSW draft distribution determination 2009-10 to 2013-14,* 21 November 2008.

4 Alternative methods to adjust for inflation

Alternative approaches to adjusting for inflation fall into two broad categories: non-market based adjustments, such as economist forecasts of inflation, and market-based adjustments such as yields on a wider range of indexed securities or the use of inflation swaps. This chapter outlines these options. In then discusses IPART's preferred option – the use of inflation swaps – in more detail.

4.1 Non-market or market-based adjustments of inflation?

One option for IPART would be to adopt economists' forecasts of inflation. This forecast would then be used to convert the nominal WACC to a real WACC. Clearly the AER's recent decisions set an important precedent for the use of inflation forecasts. Such precedents rightly have an important impact of regulatory practice. In these decisions the AER used the RBA's short term inflation forecasts and then the mid-point of the current inflation target (2.5 per cent) for the remaining period. As the latter applies for 8 of the 10 years, changes in the short term forecasts have a small effect on the average inflation rate. For example, for the SP Ausnet decision in January 2008 the 10-year inflation rate was 2.6 per cent. Economic prospects for the short to medium term have altered substantially since then but the 10-year inflation rate calculated by this approach is still 2.45. This comparative stability can be seen as a strength or weakness of the approach.

The market-based adjustments for inflation are based on the relative yields of nominal and indexed securities or the cost of other financial instruments, such as inflation swaps, that can be used to fix future payments or revenues in real terms. Such adjustments represent a "marked-to-market inflation rate" and provide a measure of the real cost of funds.

IPART considers that in principle a market-based approach to the adjustment for inflation is to be preferred, subject to the robustness and reliability of the market data and instruments upon which it is based. This approach aligns better with the principal objective – the derivation of a market-based estimate of the real cost of funds for utilities.

An economic forecast of inflation is based on a range of inputs: recent trends in inflation, macroeconomic models, especially the relationship between economic activity and inflation, and government policies. The latter is particularly critical for medium- to long-term predictions. Since 1993[13] the RBA has attempted to use monetary policy to keep inflation within a target range of 2 to 3 per cent. The 'official' forecast of 2.5 per cent inflation over the medium- to long-term assumes that:

- the current inflation target will continue unchanged
- that the RBA will target the mid-point of the range, and
- that monetary policy will continue to be effective in achieving this target.

Economic forecasts of inflation are an input into the pricing of swaps and yields on indexed securities because of the impacts on the expectations of market participants. But it is highly likely that the marked-to-market inflation rate will vary from the economists' forecasts of inflation. As noted earlier if the economist forecast is different from the marked-to-market forecast it can lead to assumptions on real interest rates that do not match the market reality that utilities face. For example, in the current environment Commonwealth Government nominal bond yields have fallen rapidly to 4.2 percent (20 day average to 14 January). An economist forecast of 10-year inflation of 2.45 per cent would imply a real risk-free rate of 1.7 per cent, with the prospect that this could go very close to zero with further reductions in nominal interest rates. While current circumstances are extreme this highlights the risks under this approach.

In contrast, while a marked-to-market inflation rate may be more volatile, and has been higher than economist forecasts of inflation for extended periods, it provides an estimate of real interest rates that better reflects those that the utility could access in the financial markets. This does not assume that a utility will try to refinance existing debt or hedge future cash flows in this way. However, consistency between the WACC assumptions and market rates is a desirable attribute.

IPART does, however, note that a marked-to-market inflation rate is not an unbiased estimate of inflation expectations or forecast of future inflation. To the extent that market participants are risk averse, participants will accept a difference between yields on nominal and indexed bonds that is wider than their expectation of inflation or pay a higher rate on inflation swap. Discussions with market experts indicated that this was indeed the case but that the bias is difficult to estimate and variable. For the reasons set out above, IPART does not consider this to be a significant problem for the purpose of the estimation of the real cost of funds. However, it may be a significant issue for the other inflation adjustments required.

13 http://www.rba.gov.au/Speeches/2003/sp_dg_100403.html

4.2 Market-based options

Given this preference for a market-based approach and the problems with the existing methodology IPART commissioned Erik Schloegl to examine the options available. His paper proposes two methodologies that are based on:

- Commonwealth and State issued inflation-indexed bonds, and
- Australian inflation-indexed swaps.

Both methodologies derive an inflation curve (ie, estimates of average inflation rates over time) for up to 30 years ahead. However, because of the relative scarcity of inflation-indexed bonds IPART is inclined to use the inflation-indexed swap model to estimate its inflation rate for the WACC, as recommended by Eric Schloegl.

Erik Schloegl recommends that the inflation-indexed bonds may be used as a benchmark against the implied inflation rates derived from swap rates where sufficient robust data on bonds is available. This point was also made by Ian Alexander from CEPA who commented that inflation-indexed bonds may not be a perfect measure to estimate inflation rates, but they may be useful as a cross-check of the values derived from the inflation swaps.

Consistent with this advice IPART intends to use the inflation–indexed bond market as a cross-check of the inflation rate derived from the swap market in future decisions.

Section 4.2.1 summarises Erik Schloegl's proposed methodology.

4.2.1 Calculation of inflation rates implied by Australian interbank inflation swaps[14]

This methodology uses interbank zero coupon inflation-indexed swaps to derive the estimates of inflation into the future (a "forward inflation curve"). Compared to the difference between yields on real and nominal bonds, this model has a number of advantages:

- no liquidity premium needs to be considered as swaps are created when needed
- quoted swap prices are the inflation rates that can be locked-in using the inflation-indexed swap market, and
- any possible maturity can be created as swaps are over-the-counter products[15].

14 An exchange of streams of payments over time according to specified terms. The most common type is an interest rate swap, in which one party agrees to pay a fixed interest rate – or the fixed leg- in return for receiving a adjustable rate – or floating leg- from another party. http://www.investorwords.com/4838/swap.html

15 Ie, a contract between the buyer of the swap and the issuing bank.

The zero coupon inflation indexed swap model determines the expected cash flow that is needed from a fixed revenue stream (the "fixed leg of a swap") to set it equal to a revenue stream indexed by inflation (the "floating leg of the swap") for different maturities along a 30-year curve.

Firstly, the model converts the discrete swap price to a continuously compounded future inflation rate. Secondly, it constructs a 30-year inflation curve[16] by averaging the continuously compounded inflation from all previous years. This methodology is straightforward and an example is given in section 4.4.

4.3 Swap Data – sources and availability

IPART subscribes to the Bloomberg data license service. This allows IPART to retrieve daily quotes for Australian inflation-indexed swaps. These are dealer quotes, meaning that these quotes are the bids and offers for zero coupon inflation-indexed swaps obtained from a panel of banks. Swaps are over-the-counter products and not traded on the exchange. Consequently, the bid/offer spreads quoted by Bloomberg are an average for dealer quoted prices in Australia. The panel of banks that supplies these quotes includes the major Australian and international banks dealing in the Australian swap market.

The fact that a bank provides a quote on a swap does not necessarily mean that the swap has traded at this particular price. However, the bank is expected to honour the quoted bid/offer spread. Not honouring a quoted spread would certainly impact on a bank's reputation in the market. Hence, IPART considers that given the low liquidity levels in the physical inflation-indexed bond market, the bid/offer quotes on inflation-indexed swaps could provide a robust proxy of the 10-year marked-to-market inflation rate required by IPART. Currently, zero coupon inflation-indexed swaps are quoted at a 10 basis points spread, that is 10 basis points between the bid and the offer quotes.

IPART notes that the Australian inflation–linked swap market is still developing. Nevertheless, given that swaps are over-the-counter products, banks can create these contracts as they are needed. This is different from the bond market where bonds are issued in a primary market and then traded in a secondary market. Another important difference between bonds and swaps is that in a swap transaction, no principal is exchanged. This means that in financial management terms, a swap transaction uses up fewer resources than a bonds transaction.

A full list of all securities used in the inflation-indexed swap model is included in Appendix B.

16 30 years is the longest available maturity available in the market.

Box 4.1 The Australian swap market

The inflation swap market emerged in Australia in the mid 1990's. Like the indexed bond market, the inflation swap market is small compared to the overall market for interest rate swaps. In 2001, the size of the Australian market for inflation swaps was estimated to be around $500 million nominal principal outstanding, compared to $6 billion for Treasury indexed bonds and $500 billion in the nominal swaps market. Unlike the nominal swaps market, where only a small fraction of swaps have a maturity of 10 years or longer, more than half the inflation swaps outstanding in 2001 have a maturity of 10 years or longer.[17] The market for zero coupon inflation-indexed bonds in Australia is growing. For example, ICAP who is the main broker in Australia for zero coupon inflation-indexed swaps more than doubled the volume traded of these instruments in the last 18 months.

Inflation indexed swaps are over-the-counter derivatives which can be used to offset inflation risk. In Australia, they are currently mostly used

- by banks wishing to offset risk associated with assets in their debt portfolios that are exposed to inflation,[18] and
- in large infrastructure transactions such as the Reliance Rail new CityRail rolling stock project.

In a zero coupon inflation-indexed swap transaction, there is only one cash flow exchanged at maturity, the indexed principal. There are no coupon payments between the time when the contract is created and when it matures. Hence the par price – or the quoted price – is the breakeven inflation rate. The simplicity of this type of swap contract makes it the preferred instrument for modelling purposes.

4.4 The swap curve model

The swap curve model is a simple Excel model that allows the user to enter the current swap rates and then produces an expected inflation and a forward inflation (ie, the expected inflation between two future dates). IPART proposes to use the midpoint of the bid/offer spread for zero coupon inflation-indexed swaps in its modelling. The bid/offer spread quoted by a bank includes an allowance for transaction costs. IPART considers that taking the midpoint of the bid/offer spread reflects the true cost of a swap including transaction costs. The Figures below show two screen shots of the data entry of the Bloomberg inflation-indexed swap quotes (Figure 4.1) and the Excel table used to compute the inflation yield-to-maturity and forward curves (Figure 4.2) as at 25August 2008.

17 RBA, *Statement on Monetary Policy*, August 2001.

18 http://www.insto.com.au/index.php?option=com_content&task=view&id=314&Itemid=13

Figure 4.1 Zero coupon inflation-indexed swaps mid-point quotes, 4 February 2009

	A	B	C	D	E	F	G
1	**Date**	**Security**	**PX_Mid**	**Security**	**PX_Mid**	**Security**	**PX_Mid**
2	4/02/2009	AUSWIT1 Curncy	1.32	AUSWIT2 Curncy	1.6	AUSWIT5 Curncy	1.98
3							

Note: IPART's own analysis.

Data source: Bloomberg.

Figure 4.2 Implied inflation to maturity 4, February 2009

AP3 =SUM($Q3:Z3)/AP$2

	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP
1		Implied inflation to maturity (continuously compounded)									
2	Years to maturity	1	2	3	4	5	6	7	8	9	10
3	Inflation rate	1.31%	1.59%	1.73%	1.85%	1.96%	2.04%	2.15%	2.19%	2.24%	2.31%

Note: IPART's own analysis.

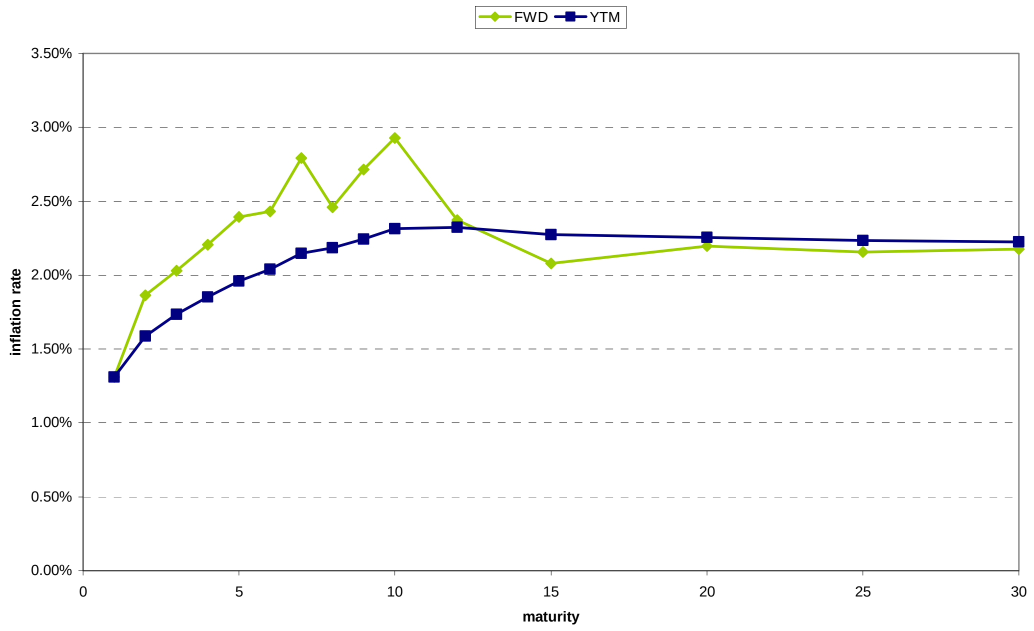
Figure 4.1 shows an extract of the data entries from the Bloomberg zero coupon inflation-indexed swap data. For example, the first security, AUSWIT 1 Curncy, is the 1-year inflation swap, quoted for 4 February 2009. The mid-point quote on that date was 1.32 per cent. This means that on 4 February 2009, the market close mid-point quote between bid and offer prices of a panel of Australian and international banks operating in Australia was an inflation rate of 1.32 per cent. The last security in the screen shot is the 5-year inflation swap (AUSWIT 5 Curncy)[19]. On 4 February 2009, inflation was priced at 1.705 per cent.

Figure 4.2 shows a screenshot on how the swap rates are converted into an inflation swap curve. This is simply done by computing the continuously compounded forward yields of the relevant swaps. For example on Figure 4.2, the formula in cell AO3 sums the continuously compounded swap rates for swaps from 1 to 10 years maturity and then divides this sum by the number of years the model is estimating the expected inflation for, in this case, 10 years.

Figure 4.3 shows the yield-to-maturity and the forward curves derived from the model for 4 February 2009.

19 Appendix 2 provides all the Bloomberg tickers of the securities used in the inflation indexed swap model.

Figure 4.3 Term structure of inflation on 4 February 2009



Note: IPART's own analysis .

Data source: Bloomberg.

Figure 4.3, shows expected inflation rates (the inflation term structure) calculated on 4 February 2009 in two forms, implied inflation yield-to-maturity and the more disaggregate (and therefore less smooth) forward inflation curve. The YTM curve, or yield-to-maturity, gives the expected average inflation for a chosen maturity. The FWD, or forward curve gives the expected inflation, viewed from today, for any particular year in the future.

For example, on 4 February 2009, the average expected inflation rate over the next ten years was 2.31 per cent (YTM curve) and the expected inflation viewed from 5 February 2009 in the tenth year is 2.93 per cent (FWD curve).

This model allows the regulator to use a marked-to-market estimate of expected inflation for any maturity between 1 and 30 years.

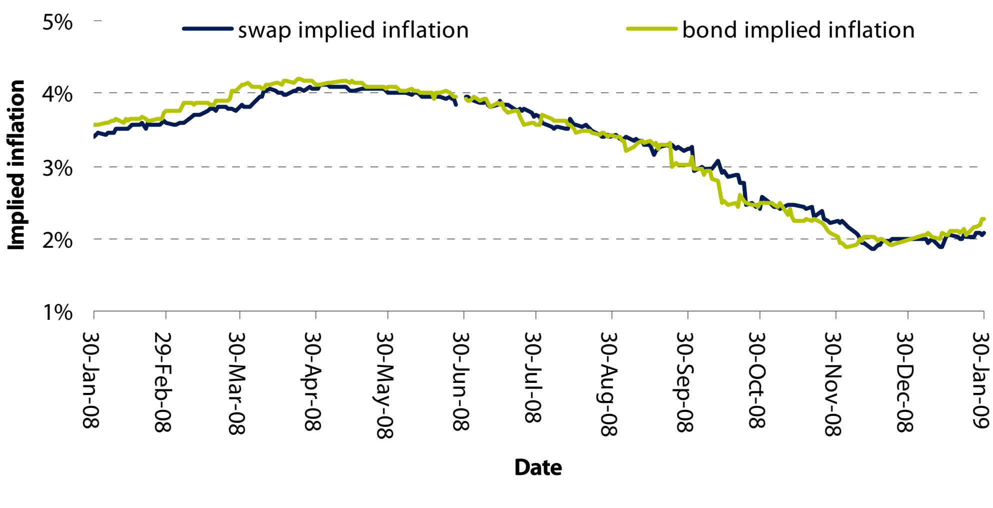
5 Comparing the methodologies

Following the methodologies discussed in the previous section, this section aims to present and compare the estimates from the proposed methodology based on zero coupon inflation-indexed swap rates and IPART's current methodology.

5.1 How do the two methodologies compare?

Continuous zero coupon inflation-indexed swap rates are available from Bloomberg[20] since early 2008. IPART has compared the 10 years inflation rates derived from the swap curve to those derived from its traditional methodology using the breakeven inflation rate between the nominal and risk-free rates for a period of 1-year (Figure 5.1).

Figure 5.1 10 years swap implied inflation and bond implied inflation (2020 inflation-indexed bond) – 30 January 2008 to 4 February 2009



Note: IPART's own analysis, Breakeven inflation includes a 20 basis points scarcity adjustment.

Data source: Bloomberg, RBA.

20 Erik Schloegl uses weekly Reuters data in his paper. IPART considers using daily Bloomberg data.

Figure 5.1 shows that the 10-year swap curve inflation is very close to the inflation rate derived from the breakeven rate between the 10-year Commonwealth Government Bond rate and the 2020 Treasury indexed bond. IPART notes that there is a maturity mismatch of around 2 years. However, this is the closest bond available in the market to match a 10-year period. Using the 2015 bond, which is the next maturity available, would result in a 3-year differential between the bond used and the target maturity.

Of note in Figure 5.1 is the fact that there is no systematic discount or premium between the two curves. One of the reasons behind this may be that inflation swaps respond to demand and supply pressures in the market. For example, during times where banks are trying to hedge inflation on large transactions, the swap market implied inflation may be higher than the bond market implied inflation. During times where no large transactions occur, the swap market implied may be lower than the bond market implied inflation. The important point for the purpose of this paper is that the swap market implied inflation is the inflation rate than can be risk-managed regardless of whether it trades above or below the bond market implied inflation rate.

IPART considers that using an inflation rate that can be risk-managed in the financial market is the only inflation rate that is consistent with IPART's financial model. IPART's financial models use the cost of capital (WACC) to compensate a business for the commercial cost of funds it would occur in a competitive market. Hence, to be consistent with a commercial cost of funds, the inflation rate used to convert the nominal into a real WACC, should reflect the true value of this inflation rate at the time of a regulatory decision. This true value is the risk-manageable rate of inflation – represented by the midpoint of the bid/offer spread on zero coupon inflation-indexed swaps.

5.2 RBA announcements and marked-to-market inflation

The Secretariat has used Erik Schloegl's methodology to derive two forward inflation curves from Australian zero coupon inflation-indexed swaps on the following two RBA target cash rate announcement days:

- 3 September 2008, a 25 basis points decrease in the cash rate (Figure 5.2), and
- 8 October 2008, a 100 basis points decrease in the cash rate (Figure 5.3).

These inflation term structure derived from the zero coupon inflation-indexed swap market are then compared to the inflation rates calculated using the breakeven inflation rate between the nominal and the real bond rates. These are the "IPART 2015 and 2020 implied inflation "points in Figures 5.2 to 5.3.

IPART chose to use RBA announcements as a market event to test and compare the 10 years inflation rates derived from the zero coupon inflation-indexed swap market and the inflation-indexed bond market. While market participants will attempt to predict the RBA's decision prior to an announcement date, quotes on the day of the announcement should fully reflect any new information communicated by the RBA. Hence, in principle, the inflation rates derived from the zero coupon inflation-indexed swap market should be very close to the inflation rates derived from the inflation-indexed bond market.

Figure 5.2 Term structure of inflation on 3 September 2008[21]

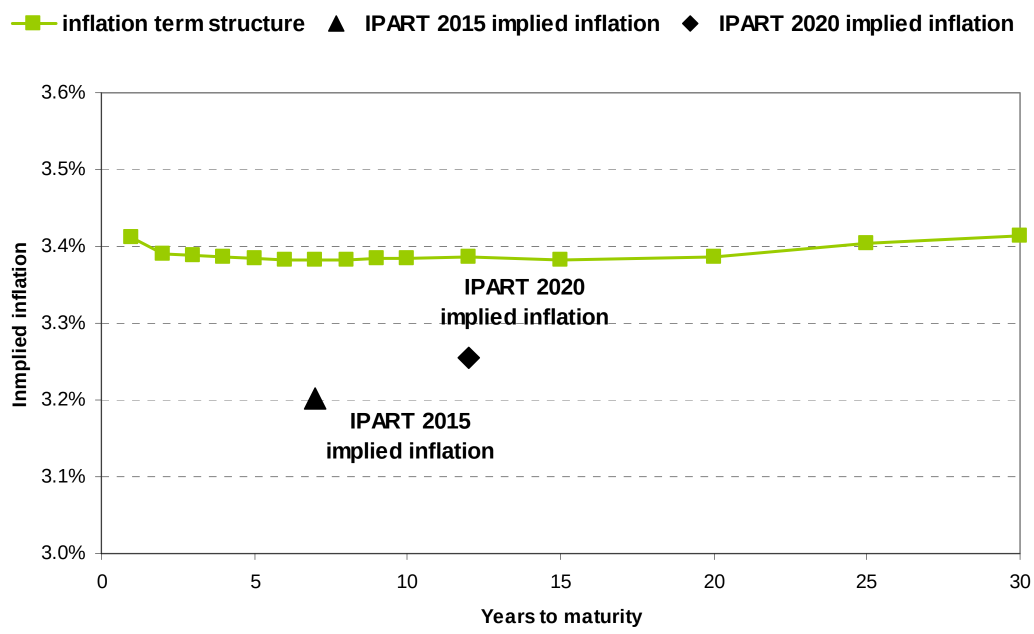


Figure 5.2 shows that when the RBA announced a decrease in the cash rate of 25 basis points, IPART's methodology of using the breakeven inflation rate yields lower inflation rates for the 7 (2015 inflation-linked bond) and 12-year (2020 inflation-linked bond) maturities than the swap market inflation rates. The differences between the new and the traditional methodology on the 3 September 2008 is between 18 and 14 basis points.

21 The 10 years inflation rate derived from IPART's traditional methodology includes a 20 basis points scarcity adjustment.

Figure 5.3 Term structure of inflation on 8 October 2008

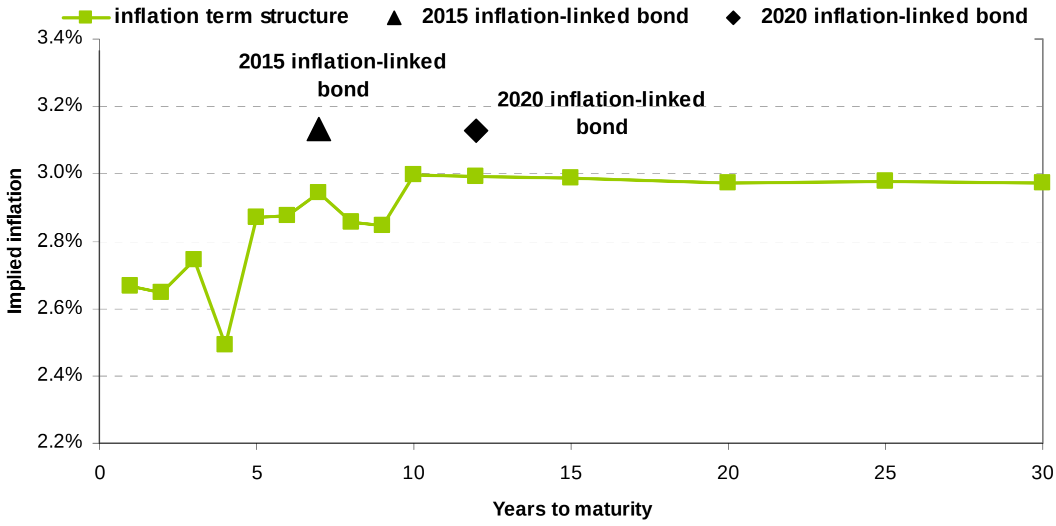


Figure 5.3 shows that when the RBA cut the cash rate by 100 basis points on 8 October 2008, IPART's methodology of using the breakeven inflation rate yields a higher inflation rate for the 7 (2015 inflation-linked bond) and the 12-year (2020 inflation-linked bond) maturities than the swap market inflation rates. The difference between the new and the traditional methodologies on the 3 September 2008 is between 19 and 14 basis points in absolute values. Of note is the differential in the yield between say, the 3-year and the 4-year maturities and the four- and the 5-year maturities of the expected swap curve. IPART has investigated this and found that there seems to be substantial volatility in the shorter maturities. While this volatility does not seem to occur for the 10-year maturity, it is nevertheless an indicator that an average rather than a point estimate of the yield is an important step in smoothing out potential volatility. IPART has had suggestions from finance professionals that a three months fortnightly or three months weekly average is more appropriate than a 20-day average to smooth out volatility for a 10 years maturity.

IPART notes that the volatility of the 10 years zero coupon inflation-indexed swap rates are similar to the volatility of the 10 years breakeven inflation rate derived from the bond market (Figure 5.1). IPART computed the standard deviations for the 10 years swap and bond market implied inflation figures and found that they are very similar. For the period from 30 January 2008 to 3 February 2009:

- the 10 years swap market inflation rate's standard deviation[22] is 1.85 per cent[23], and

22 The standard deviation can be used as a measure of volatility.

23 Calculation based on continuously compounded returns.

- the 10 years bond market implied inflation rate's standard deviation is 1.89 per cent.

Given that the 10 years swap market implied inflation is less volatile than the 10 years bond market implied inflation, IPART considers that averaging zero coupon inflation-indexed swap quotes over a 20-day time period should, at least, provide the same smoothing effect on day-to-day volatility as it has provided for the 10 years bond market implied inflation.

Figures 5.2 and 5.3 indicate that Erik Schloegls' methodology yields results that are very similar to IPART's current methodology to estimate a 10 years inflation rate[24]. In the examples given, the difference is below 20 basis points and is not systematic, that is it may be higher or lower than the estimate derived using the traditional methodology.

The Bank of England noted in 2006[25], that in theory, inflation curves derived from inflation swaps and indexed-linked bonds should be identical, but in practice there can be differences. In the market of indexed securities, these differences may be caused by:

- A lack of securities in either market. For example, it should be easier to price long term expected inflation from swaps for missing maturities than from indexed bonds where only two maturities in excess of 5 years are available.
- If there are no arbitrage opportunities, relative demand and supply in each market would determine pricing and might cause expected inflation rates to be different in one market versus the other.

IPART is currently adjusting the breakeven inflation rate by 20 basis points to adjust for supply shortages. Using zero-coupon inflation-indexed swaps would make such an adjustment redundant.

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24 The IPART 10 years inflation rates used in Figures 1 to 3 do include a 20 basis points scarcity adjustment in the real risk-free rate.

25 Bank of England, *New Information from inflation swaps and indexed-linked bonds*, Bank of England Quarterly Bulletin, Spring 2006.

Appendices

A Derivation of methodologies

EXTRACTING THE MARKET–IMPLIED TERM STRUCTURE OF FORWARD INFLATION

ERIK SCHLÖGL

Executive Summary. This report discusses ways of extracting the information about the "market consensus" on anticipated future inflation from the prices of traded financial instruments. Using such a "term structure of inflation" to value inflation–contingent future cashflows will — by construction — appropriately reflect a market equilibrium inflation risk premium. Furthermore, to calculate a real interest rate faced by a particular entity, including any credit risk premium, one would subtract the implied forward inflation rate from the nominal interest rate this entity has to pay when borrowing funds.

In order to avoid existing problems with implying a forward inflation curve from inflation–linked bonds (ILBs), IPART should move to using quotes from the inflation swap market as the primary market data input. This addresses the lack of new (longer maturity) issues of Commonwealth government ILBs and the scarcity premium resulting from the limited supply of these bonds.

1. Objective

The goal is to extract "market–consensus" inflation expectations from traded instruments such as inflation–linked bonds or inflation swaps to inform the decision–making process of IPART. This approach has several advantages:

- It is *objective*, as the inflation expectations are determined from market prices in a transparent and reproducible manner.
- It is *forward–looking*, in the sense of the well–established economic principle that markets aggregate all available information and expectations of market participants into market prices, as opposed to econometric forecasts which attempt to extrapolate historical data into the future.
- It results in an inflation forecast that is *marked–to–market*, in the sense that this the forecast that has been priced into market–traded instruments and it represents the future level of inflation that could be risk–managed (up to transaction costs) using these instruments. In a manner similar to the way one can manage the risk of future movements in interest rates by locking in the current forward rate, the methodology considered here results in a term structure of forward inflation rates.

The last point in particular makes a market–based methodology critical for an appropriate accounting of inflation risks. Essentially, the argument for using a market–implied forward inflation curve to evaluate inflation–contingent future cashflows mirrors the established practice of calculating the net present value of known future cashflows by discounting using the term structure of interest rates currently observed in the market (as opposed to applying discounting based on some statistical prediction of future interest rates).

By the same token, the interpretation of implied forward inflation rates as "inflation expectations" is analogous to the risk–neutral pricing paradigm put on a rigorous foundation by Harrison and Kreps (1979) and Harrison and Pliska (1981) and now considered

Date. February 9, 2009.

standard in the finance literature. Thus the implied forward inflation rates will *not* be unbiased estimates of future inflation, but rather incorporate a market equilibrium inflation risk premium which *will be appropriately reflected* when using the inflation curve to evaluate inflation–contingent future cashflows.

2. Addressing data availability

The above points are powerful arguments in favour of a market–based methodology; however, market imperfections such as credit risk and liquidity require a number of issues to be addressed in order to make it practicable. The requisite assumptions and their implications are made explicit in the next section. Foremost, however, stands the issue of data availability.

The availability of inflation–linked bonds (ILBs) in the Australian market is quite limited. The Commonwealth government is not currently issuing ILBs and has not done so for some time. Thus the longest available bond of this type matures in 2020, which for the purposes of IPART may be an insufficiently distant time horizon on the forward inflation curve. Furthermore, it has been suggested[1] that the lack of sufficient supply of both nominal and inflation–linked Commonwealth government securities has led to a scarcity premium on these securities, resulting in absolute and relative biases when nominal and real interest rates are implied from the prices of these instruments.

Although Australian state governments have also issued inflation–linked bonds, with the longest maturity of e.g. the NSWTC bonds being 2035, the pricing data made available by the data provider Bloomberg is quite sparse, i.e. prices are not available on a daily basis. Consequently, it does not appear advisable to rely on this data alone when extending the forward inflation curve beyond 2020. For ILBs issued by corporate entities such as Sydney Airport Finance, Westpac Banking Corporation or Envestra Victoria Ltd, the situation is even worse, with almost no pricing data available through Bloomberg.

Australian zero coupon inflation–indexed swaps (ZCIIS) represent a viable alternative source of market information. Weekly contributor–polled rates are quoted on the Reuters page CPISWAPREF, an example of which for rates on 28 August 2008 is given in Figure 1. Note that arguably no scarcity premium applies in this market, as these are zero net supply derivative financial instruments, where contracts can be created as required. Furthermore, the time horizon for the implied forward inflation curve is always 30 years out from the quote date. Thus one can proceed by using the ZCIIS as the primary data source. Comparison with the Commonwealth and (where available) state government issued ILBs can serve to quantify the "scarcity premium" due to the limited supply of these instruments.

3. Methodology

3.1. **The forward inflation curve implied by Australian government and semi-government inflation–linked bonds (ILBs).** The Australian CPI figure is released quarterly, on the 4$^{\text{th}}$ Wednesday of the months April, July, October and January, for the quarters ending in the preceding march, June, September and December, respectively. Denote these dates by τ_i.

[1]The submissions of Hird, Grundy and Anderson (2007) and Hird, Grundy and Anderson (n.d.) raise this issue.

07:14 29AUG08 AUST INFLATION SWAP INTERBANK AVERAGE AU18992 CPISWAPREF

SWAP VALUATION RATES AT 16:30 28/8/08 3YR 94.33 10YR 94.265

AVERAGE:

3YR	5YR	7YR	10YR	12YR	15YR	20YR	25YR	30YR
3.42/52	3.42/52	3.41/51	3.41/51	3.41/51	3.42/52	3.42/52	3.42/52	3.42/52

ELIGIBLE CONTRIBUTORS RATES

3YR	5YR	7YR	10YR	12YR	15YR	20YR	25YR	30YR
3.40/50	3.40/50	3.40/50	3.40/50	3.40/50	3.42/52	3.42/52	3.42/52	3.43/53
3.45/55	3.50/60	3.49/59	3.43/53	3.43/53	3.42/52	3.40/50	3.42/52	3.44/54
3.40/50	3.40/50	3.40/50	3.40/50	3.40/50	3.40/50	3.40/50	3.40/50	3.40/50
3.42/52	3.42/52	3.42/52	3.41/51	3.41/51	3.41/51	3.41/51	3.40/50	3.40/50
3.41/51	3.41/51	3.41/51	3.42/52	3.42/52	3.42/52	3.42/52	3.42/52	3.43/53
3.38/48	3.38/48	3.38/48	3.38/48	3.37/47	3.37/47	3.36/46	3.35/45	3.35/45
3.42/52	3.40/50	3.40/50	3.43/53	3.44/54	3.45/55	3.45/55	3.45/55	3.45/55
3.48/58	3.48/58	3.47/57	3.43/52	3.43/53	3.43/53	3.43/53	3.43/53	3.44/54
3.46/56	3.40/50	3.37/47	3.42/52	3.43/53	3.43/53	3.44/54	3.45/55	3.45/55
3.40/50	3.40/50	3.40/50	3.41/51	3.41/51	3.40/50	3.40/50	3.40/50	3.40/50
3.43/53	3.43/53	3.42/52	3.40/50	3.40/50	3.43/53	3.43/53	3.43/53	3.43/53
3.42/52	3.42/52	3.41/51	3.40/50	3.40/50	3.42/52	3.44/54	3.44/54	3.44/54

BANKS CONTRIBUTING TODAY:ABN ANZ BNP CBA CITI DEU GS JP MBL NAB RBS UBS

FIGURE 1. Weekly contributor–polled ZCIIS rates

The inflation–linked bonds[2] pay coupons quarterly, on the 20$^{\text{th}}$ day of the months May, August, November and February. Denote these dates by T_i, such that $T_i - \tau_i$ is the year fraction between the end of a CPI quarter and the closest following coupon payment date, i.e. if τ_i is the end of March 2008, T_i is 20 May 2008.

Denote by $K(T_i)$ the principal of the inflation–linked bond at time T_i. At each time T_i, the principal is increased by

$$K(T_i) = K(T_{i-1})(1 + p(T_i)) \tag{1}$$

where

$$p(T_i) = \frac{1}{2}\left(\frac{\text{CPI}(\tau_{i-1})}{\text{CPI}(\tau_{i-3})} - 1\right) \tag{2}$$

Consider now an inflation–linked bond issued at time T_0 and maturing at time T_N, with a coupon of $c\%$. Then the coupon payments at times T_i, $1 \leq i \leq N$, are given by

$$\frac{c}{4}\frac{K(T_i)}{100}$$

and at time T_N the principal $K(T_N)$ is also paid to the bond holder. Assume that $K(T_0) = 100\%$.

Denote by $I_C(t)$ the market price of a Commonwealth government inflation–linked bond. Assume that the quoted price does not include accrued interest (as per the usual market convention), so $I_C(t)$ is the sum of the quoted price and accrued interest, given by

$$\frac{c}{4}\frac{K(T_{\eta(t)})}{100}\frac{t - T_{\eta(t)-1}}{T_{\eta(t)} - T_{\eta(t)-1}}$$

where $T_{\eta(t)}$ is the next date in the tenor structure $\{T_i\}$ after t.

Note that $K(T_i)$ is already known at time θ_{i-1}, where θ_{i-1} denotes the CPI release date corresponding to the quarter ending at time τ_{i-1}. Thus at time t some future cashflows of

[2]The specification of ILBs presented here is documented in Commonwealth of Australia (1995).

the ILB are already known, i.e. all cashflows at times $\{T_j : T_j > t, \theta_{j-1} \leq t\}$. The present value of these known payments is

$$\sum_{\{T_j:T_j>t,\theta_{j-1}\leq t\}} \frac{c}{4}\frac{K(T_j)}{100} B_C(t,T_j) \tag{3}$$

where $B_C(t,T_j)$ is the relevant discount factor implied by the standard (non-IL) Commonwealth government bonds. These are the correct discount factors to use for known cashflows of Commonwealth government ILBs if we make

1. **Assumption.** *The scarcity premium (if any) is identical in the markets for Commonwealth government ILBs and non-IL bonds.*

Thus regardless of the validity of Assumption 1, any absolute bias due to scarcity of Commonwealth government bonds cancels out and is not an issue. If Assumption 1 is lifted, the *relative* bias of ILBs versus non–IL bonds will have an effect, which can be quantified by comparing the forward inflation curves implied by ILBs versus those implied by inflation swaps.

Denote by $\hat{I}_C(t)$ the market price of the ILB with the present value of known future cashflows removed. This must then equal the expected (under the appropriate probability measure) discounted value of the unknown future payments.

2. **Assumption.** *The scarcity premium $\lambda_C(t)$ for Commonwealth government bonds is deterministic (though possibly time–dependent).*[3]

We can write under the spot martingale measure

$$\begin{aligned} \hat{I}_C(t) &= E\left[\frac{c}{400}\sum_{\{T_j:t<T_j\leq T_N,\theta_{j-1}>t\}} K(T_j)\exp\left\{-\int_t^{T_j}(r_N(s)+\lambda_C(s))ds\right\}\middle|\mathcal{F}_t\right] \\ &\quad + E\left[K(T_N)\exp\left\{-\int_t^{T_N}(r_N(s)+\lambda_C(s))ds\right\}\middle|\mathcal{F}_t\right] \end{aligned} \tag{4}$$

where $r_N(s)$ is the riskfree nominal rate of interest (with continuous compounding). With $\lambda_C(s)$ deterministic, we can change measures to the appropriate forward probabilities and write

$$\begin{aligned} \hat{I}_C(t) &= B_C(t,T_N)E_{T_N}\left[K(T_N)|\mathcal{F}_t\right] \\ &\quad + \frac{c}{400}\sum_{\{T_j:t<T_j\leq T_N,\theta_{j-1}>t\}} B_C(t,T_j)E_{T_j}[K(T_j)|\mathcal{F}_t] \end{aligned} \tag{5}$$

where E_{T_j} denotes the expectation operator under the time T_j forward measure. This allows us to bootstrap the $E_{T_j}[K(T_j)|\mathcal{F}_t]$ from market prices of Commonwealth government ILBs without explicit recourse to the (possibly unobservable) "true" riskfree nominal rate of interest $r_N(s)$ and the scarcity premium $\lambda_C(s)$.

In order to extend the inflation term structure as far as possible, we include state government bonds. For each issuer, we make analogous assumptions to Assumptions 1 and 2 (though the scarcity premium is permitted to be different for each issuer). This

[3]It is theoretically possible to model a stochastic scarcity premium, but such a model would be difficult to calibrate and is unlikely to add much of relevance to the problem at hand.

allows us to value the known future cashflows in each state ILB analogously to (3) by

$$\sum_{\{T_j:T_j>t,\theta_{j-1}\leq t\}} \frac{c}{4}\frac{K(T_j)}{100}B_S(t,T_j) \tag{6}$$

to arrive at the present value $\hat{I}_S(t)$ of unknown future cashflows, where $B_S(t,T_j)$ is the relevant discount factor implied by the standard (non-IL) bonds of the same issuer.

Typically, state government bonds trade at a spread to Commonwealth government bonds. In order to arrive at an analogous expression to (5), i.e.

$$\begin{aligned} \hat{I}_S(t) &= B_S(t,T_N)E_{T_N}\left[K(T_N)|\,\mathcal{F}_t\right] \\ &\quad + \frac{c}{400}\sum_{\{T_j:t<T_j\leq T_N,\theta_{j-1}>t\}} B_S(t,T_j)E_{T_j}[K(T_j)|\mathcal{F}_t] \end{aligned} \tag{7}$$

we need **one** of the following two assumptions, either

3. **Assumption.** *The spread between Commonwealth and state government bonds is solely due to scarcity (not credit risk).*

or (more realistically)

4. **Assumption.** *Under the forward probability measures, the CPI, state government credit default, and recovery on state government debt in default, are all independent of each other.*

Thus we now have a term structure of expected values

$$E_{T_j}[K(T_j)|\mathcal{F}_t] = E_{T_j}\left[\prod_{i=1}^{j}(1+p(T_i))\middle|\,\mathcal{F}_t\right] \tag{8}$$

The definition (2) of the $p(T_i)$ in terms of overlapping two–quarter periods means that $K(T_j)$ is a quite nonlinear function of the CPI. Strictly speaking, there is no straightforward expression linking (8) to the expected CPI or expected inflation. However, for all intents and purposes $K(T_j)$ evolves like a smoothed CPI, so one is justified in setting

$$E_{T_j}[CPI(\tau_{j-1})|\mathcal{F}_t] \approx E_{T_j}[K(T_j)|\mathcal{F}_t] \tag{9}$$

Following Jarrow and Yildirim (2003), the CPI can be interpreted as an exchange rate between a "real" and a "nominal" economy. Define $R(t,T)$ as the discount factor in the real economy, i.e.

$$R(t,T) = \exp\left\{-\int_t^T f_R(t,s)ds\right\} \tag{10}$$

where the f_R are the forward real interest rates. Analogously, $B(t,T)$ is the nominal discount factor

$$B(t,T) = \exp\left\{-\int_t^T f_N(t,s)ds\right\} \tag{11}$$

Ignoring any convexity correction, we can write

$$
\begin{aligned}
(12)\qquad E_{T_j}[K(T_j)|\mathcal{F}_t] &\approx E_{\tau_{j-1}}[\mathrm{CPI}(\tau_{j-1})|\mathcal{F}_t]\\
&= E_{\tau_{j-1}}\left[\frac{\mathrm{CPI}(\tau_{j-1})R(\tau_{j-1},\tau_{j-1})}{B(\tau_{j-1},\tau_{j-1})}\right|\left.\mathcal{F}_t\right]\\
&= \frac{\mathrm{CPI}(t)R(t,\tau_{j-1})}{B(t,\tau_{j-1})}\\
&= \mathrm{CPI}(t)\exp\left\{-\int_t^{\tau_{j-1}}(f_R(t,s)-f_N(t,s))ds\right\}\\
&= \mathrm{CPI}(t)\exp\left\{\int_t^{\tau_{j-1}}f_I(t,s)ds\right\}
\end{aligned}
$$

where $f_I(t,s) = f_N(t,s) - f_R(t,s)$ is the implied forward inflation rate. Thus we can determine f_I directly from the market data without recourse to f_R or f_N.

To calculate a real interest rate faced by a particular entity, including any credit risk premium that this entity has to pay, one would subtract the implied forward inflation rate from the nominal interest rate this entity has to pay when borrowing funds. If desired, this could be adjusted to a riskless real interest rate by subtracting the credit spread obtained from credit default swaps or from an average credit spread faced by obligors of similar credit rating as the entity in question.

If desired, (12) can be adjusted to include a convexity correction. This adjustment is model–dependent. Define

$$
(13)\qquad \hat{C}_{\tau_{j-1}}(t) = \frac{\mathrm{CPI}(t)R(t,\tau_{j-1})}{B(t,\tau_{j-1})}
$$

$\hat{C}_{\tau_{j-1}}(t)$ under the time τ_{j-1} forward measure is a martingale (in any arbitrage–free model). Specifically, in the Jarrow and Yildirim (2003) model, the dynamics of $\hat{C}_{\tau_{j-1}}(t)$ are

$$
(14)\qquad \frac{d\hat{C}_{\tau_{j-1}}(t)}{\hat{C}_{\tau_{j-1}}(t)} = \left(\sigma_{\mathrm{CPI}}(t) - \left(\int_t^{\tau_{j-1}}\sigma_R(t,s)ds\right) + \int_t^{\tau_{j-1}}\sigma_N(t,s)ds\right)dW_{\tau_{j-1}}(t)
$$

where $W_{\tau_{j-1}}$ is a (vector–valued) Brownian motion under the τ_{j-1} forward probability measure, $\sigma_R(t,s)$ is the volatility of the forward real interest rate, i.e.

$$
-\int_t^{\tau_{j-1}}\sigma_R(t,s)ds
$$

is the volatility of $R(t,\tau_{j-1})$; and analogously $\sigma_N(t,s)$ is the volatility of the forward nominal interest rate. The Radon/Nikodym derivative to change from the τ_{j-1} forward measure to the T_j forward measure is given by

$$
\begin{aligned}
\left.\frac{dP_{T_j}}{dP_{\tau_{j-1}}}\right|_{\mathcal{F}_t} &= \frac{B(0,\tau_{j-1})B(t,T_j)}{B(t,\tau_{j-1})B(0,T_j)}\\
(15)\qquad &= \exp\left\{-\int_0^t\int_{\tau_{j-1}}^{T_j}\sigma_N(u,s)ds dW_{\tau_{j-1}}(u) - \frac{1}{2}\int_0^t\left(\int_{\tau_{j-1}}^{T_j}\sigma_N(u,s)ds\right)^2du\right\}
\end{aligned}
$$

Therefore, by Girsanov's theorem,

$$
(16)\qquad dW_{T_j}(t) = dW_{\tau_{j-1}}(t) + \left(\int_{\tau_{j-1}}^{T_j}\sigma_N(t,s)ds\right)dt
$$

and (14) becomes

$$\frac{d\hat{C}_{\tau_{j-1}}(t)}{\hat{C}_{\tau_{j-1}}(t)} = -\left(\sigma_{\mathrm{CPI}}(t) - \left(\int_t^{\tau_{j-1}} \sigma_R(t,s)ds\right) + \int_t^{\tau_{j-1}} \sigma_N(t,s)ds\right)\left(\int_{\tau_{j-1}}^{T_j} \sigma_N(t,s)ds\right)dt$$

$$(17) \qquad + \left(\sigma_{\mathrm{CPI}}(t) - \left(\int_t^{\tau_{j-1}} \sigma_R(t,s)ds\right) + \int_t^{\tau_{j-1}} \sigma_N(t,s)ds\right)dW_{T_j}(t)$$

and

$$\begin{aligned} & E_{T_j}[K(T_j)|\mathcal{F}_t] \\ \approx\ & E_{T_j}[\mathrm{CPI}(\tau_{j-1})|\mathcal{F}_t] \\ =\ & E_{T_j}[\hat{C}_{\tau_{j-1}}(\tau_{j-1})|\mathcal{F}_t] \\ =\ & \exp\left\{-\int_t^{\tau_{j-1}}\left(\sigma_{\mathrm{CPI}}(u) - \left(\int_u^{\tau_{j-1}} \sigma_R(u,s)ds\right) + \int_u^{\tau_{j-1}} \sigma_N(u,s)ds\right)\left(\int_{\tau_{j-1}}^{T_j} \sigma_N(u,s)ds\right)du\right\} \\ & \cdot\ \mathrm{CPI}(t)\exp\left\{\int_t^{\tau_{j-1}} f_I(t,s)ds\right\} \end{aligned}$$

Thus the convexity correction requires the calibration/estimation of the volatilities and covariation of the CPI as well as of the nominal and real discount factors. This is a substantial task by itself; one that can be set aside for future consideration after the initial methodology has been implemented.

3.2. **The forward inflation curve implied by Australian interbank inflation swaps.** Weekly contributor–polled rates for interbank zero coupon inflation–indexed swaps (ZCIIS) are quoted on the Reuters page CPISWAPREF. Following Brigo and Mercurio (2006), consider a ZCIIS maturing at time T_M, M years from T_0 (now), i.e. $T_M - T_0 = M$. At time T_M, on a notional amount N, a fixed amount

$$(18) \qquad N[(1+K)^M - 1]$$

is exchanged for the floating amount

$$(19) \qquad N\left[\frac{\mathrm{CPI}(T_M)}{\mathrm{CPI}(T_0)} - 1\right]$$

where K is the ZCIIS rate fixed at inception of the swap in T_0. K is fixed such that the net present value of the swap at inception is zero; thus

$$(20) \qquad E_{T_M}\left[\frac{\mathrm{CPI}(T_M)}{\mathrm{CPI}(T_0)}\right] = (1+K)^M$$

and by the martingale property of the forward CPI under the time T_M forward probability measure

$$\begin{aligned} (21) \qquad (1+K)^M &= E_{T_M}\left[\frac{\mathrm{CPI}(T_M)}{\mathrm{CPI}(T_0)}\right] \\ &= E_{T_M}\left[\frac{\mathrm{CPI}(T_M)R(T_M,T_M)}{\mathrm{CPI}(T_0)B(T_M,T_M)}\right] \\ &= \frac{R(T_0,T_M)}{B(T_0,T_M)} \\ &= \exp\left\{\int_{T_0}^{T_M} f_I(T_0,s)ds\right\} \end{aligned}$$

and as with ILBs in the previous section, the forward inflation rate can be implied from market quotes without any reference to the "true" riskfree interest rates, real or nominal.

Note that arguably no scarcity premium applies in this market, as it is a zero net supply derivative market, where contacts can be created as required.

4. Implementation

As Section 3.2 shows, the rates quoted in the Australian interbank market for zero coupon inflation–indexed swaps are related to implied forward inflation rates in a very straightforward manner, making it easy to extract a forward inflation curve. This is illustrated in the file

`hist_102308_cpi_swaps_implied_fwd_inflation.xls`

If we extract the implied forward inflation in the for of continuously compounded forward rates, the implied inflation over the first year is given by

$$f_1 = \ln(1+K_1), \tag{22}$$

over the second year by

$$f_2 = 2\ln(1+K_2) - f_1, \tag{23}$$

and in general by

$$f_j = j\ln(1+K_1) - \sum_{i=1}^{j-1} f_i. \tag{24}$$

Note that forward inflation rates covering multiple years (i.e. beyond ten years out) must be multiplied by the number of years they cover when subtracting the prior forward inflation rates in (24).

Figure 2 shows the implied inflation curve on 25 August 2008 in two representations, implied inflation yield to maturity and the more disaggregate (and therefore less smooth) forward inflation rate representation.

At times there are missing values, which must be interpolated. We choose loglinear interpolation of (inflation) discount factors, which corresponds to flat interpolation of continuously compounded forward inflation rates. Defining

$$I(T_0,T_j) = \exp\left\{-\int_{T_0}^{T_j} f_I(T_0,s)ds\right\} \tag{25}$$

we interpolate $I(T_0,T_j)$ from $I(T_0,T_{j-1})$ and $I(T_0,T_{j+1})$ by

$$\ln I(T_0,T_j) = \ln I(T_0,T_{j-1}) + \frac{T_j - T_{j-1}}{T_{j+1} - T_{j-1}}(I(T_0,T_{j+1}) - I(T_0,T_{j-1})) \tag{26}$$

5. Conclusion

The present paper has demonstrated how a viable forward inflation curve can be constructed using data from the Australian market for inflation swaps. This can be benchmarked against forward inflation extracted from prices of inflation–linked bonds, allowing the relative scarcity premium associated with these bonds to be quantified.

Ideally, the impact of the simplifying assumptions should be analysed in a simulation study (e.g. convexity correction for ILBs, but also counterparty credit risk for ZCIIS). Conducting an empirical comparison between inflation–linked bond and swap markets overseas (where these markets are more developed) can inform an evaluation of whether

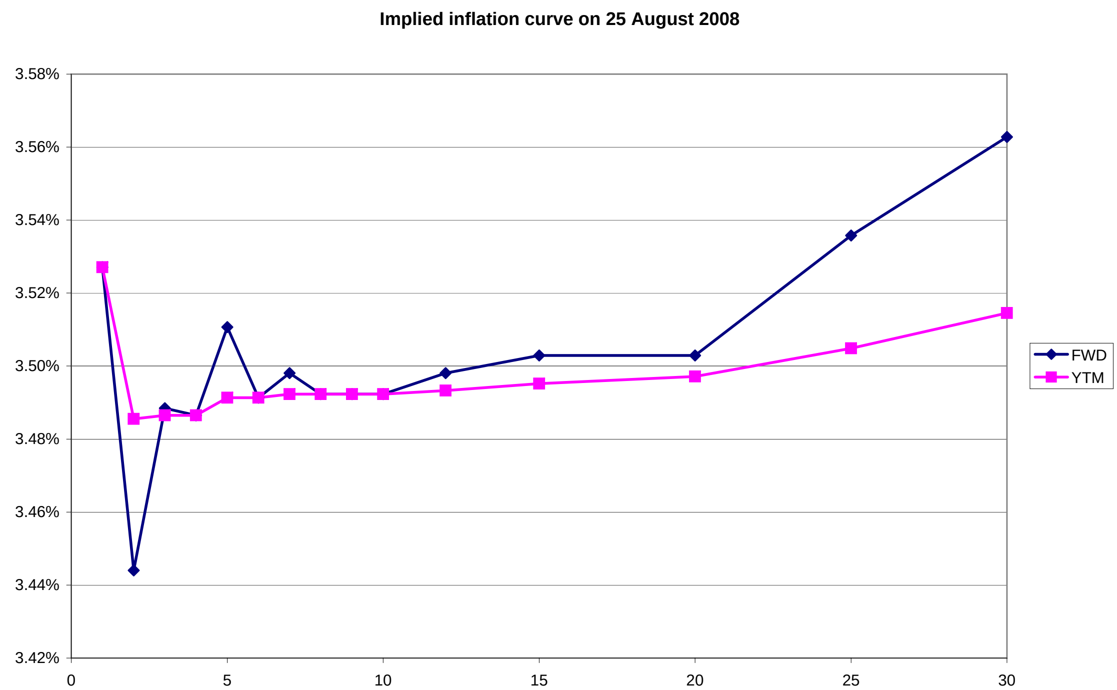

FIGURE 2. Continuously compounded implied inflation yield to maturity (YTM) and forward rates (FWD)

the Australian ZCIIS market is a reasonable benchmark for market–implied Australian forward inflation.

The proposed methodology avoids direct reference to the “true” riskfree nominal and real interest rates, using instead the appropriate nominal interest rates faced by the particular entity in question (Commonwealth government, state government or corporate). If required, the real interest rates for each entity can be calculated by subtracting the implied forward inflation rates from these nominal interest rates. The methodology is transparent, objective, forward–looking and most importantly market–driven, meaning that the value of inflation–contingent future cashflows calculated based on the forward inflation curve by construction incorporates the market–consensus inflation risk premium.

REFERENCES

Brigo, D. and Mercurio, F.: 2006, *Interest Rate Models: Theory and Practice*, 2nd edn, Springer-Verlag.

Commonwealth of Australia: 1995, Treasury indexed bonds to be offered for sale by periodic tender redeemable at maturity, Prospectus.

Harrison, J. M. and Kreps, D. M.: 1979, Martingales and arbitrage in multiperiod securities markets, **20**, 381–408.

Harrison, J. M. and Pliska, S. R.: 1981, Martingales and stochastic integrals in the theory of continuous trading, *Stochastic Processes and their Applications* **11**, 215–260.

Hird, T., Grundy, B. and Anderson, S.: 2007, Absolute bias in (nominal) Commonwealth Government Securities, *Technical report*, NERA Economic Consulting.

Hird, T., Grundy, B. and Anderson, S.: n.d., Relative bias in indexed CGS bonds, *Technical report*, NERA Economic Consulting.

Jarrow, R. and Yildirim, Y.: 2003, Pricing Treasury inflation protected securities and related derivatives using an HJM model, *Journal of Financial and Quantitative Analysis* **38**(2), 337–358.

B Bloomberg identifiers

Table B.1 Inflation swap Bloomberg tickers

Security	Security ticker
1-year inflation swap	AUSWIT1 Curncy
2-year inflation swap	AUSWIT2 Curncy
3-year inflation swap	AUSWIT3 Curncy
4-year inflation swap	AUSWIT4 Curncy
5-year inflation swap	AUSWIT5 Curncy
6-year inflation swap	AUSWIT6 Curncy
7-year inflation swap	AUSWIT7 Curncy
8-year inflation swap	AUSWIT8 Curncy
9-year inflation swap	AUSWIT9 Curncy
10-year inflation swap	AUSWIT10 Curncy
12-year inflation swap	AUSWIT12 Curncy
15-year inflation swap	AUSWIT15 Curncy
20-year inflation swap	AUSWIT20 Curncy
25-year inflation swap	AUSWIT25 Curncy
30-year inflation swap	AUSWIT30 Curncy

Source: Bloomberg.

Glossary

Breakeven inflation rate: The Breakeven inflation rate is the difference between the nominal and the real risk-free rate. IPART calculates the breakeven inflation rate using the Fisher equation.

Implied inflation: Implied inflation is the inflation rate that can be derived from financial market instruments. For example the difference between the nominal and the real Commonwealth Government bond rates is the Commonwealth Government bond market implied inflation.

Inflation term structure: A term structure of inflation represents the inflation rates over a series of maturities. In this paper, the term structure ranges from 1 year to 30-years maturity.

Inflation-indexed bonds: Inflation-indexed bond are bonds which have their coupons and the principal indexed by inflation. There are various methods used to adjust for inflation, but in general the inflation adjustment is lagged by several months.

Forward rate: A forward rate is an inflation (or interest) rate for a specified future period. For example the five to six years forward inflation is the inflation rate starting in five years from now and ending in six years.

Marked-to-market: Refers to the financial markets valuation of financial securities – for example in this paper the zero coupon inflation –indexed swaps. For example, the bid/offer quotes on financial securities are said to be marked-to-market as these are the quotes banks are willing to trade at based on current valuations.

Yield-to-maturity: The average inflation rate over a specified period of time.

Zero coupon inflation-indexed swap: Transaction that involves the exchange of the difference between a fixed and a floating inflation rate indexed principal at the end of a specified period. The floating rate indexed component is indexed by the actual inflation rate and the fixed component is indexed by the par (or the swap rate) rate of inflation.